Nonequilibrium unsteady thermodynamic and spatio-temporal properties of ground source heat pumps

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Presentation to cover

- Motivation
- Problem statement
- Dimensionless parameters
- Similarity problems for VGS
- Energy analysis of VGS systems
- Conclusions
Motivation

Sizing of BHE – a vital issue in the design of VGS

- Conventional engineering theories of BHE are based on the assumptions of constancy of temperature or heat fluxes along a borehole. The assumptions are grounded on Kelvin’s theory of heat sources: unsteady, one dimensional, self similar solution
- The assumptions mean that the amount of extracted heat is directly proportional to the length of the BHE

There is no justification for these assumptions for actual operating conditions of VGS

The conventional theories do not deliver on optimization of the BHE length
Goals

- To elaborate an unsteady phenomenological thermodynamic theory of VGS as an integrated system combining soil, BHE, and GSHP in a nonequilibrium thermodynamic system.

- To study fundamental laws of energy exchange in VGS in order to develop new approaches to construction of energy-efficient schemes for VGS using similarity methods and laboratory modeling.
Thermodynamic scheme for a VGS

\[ E = c_{pb} \rho \int_S [T_b(t, r, x_0, k_{sb}, \alpha_{sb}) - T_b(t, r, x_0 = 0, k_{sb}, \alpha_{sb})]V(r)ds \]

Soil – source of low temperature energy

\[ T_0, \alpha_S, k_S \]

BHE

\[ L, R, k_b, \alpha_b, \dot{V}, h_w, k_w, \alpha_w, h_{gr}, k_{gr}, \alpha_{gr} \]

Evaporator

\[ T_{inlet}, \Delta T \]

Compressor

\[ T_{cond} \]

Energy Quality

\[ COP = \frac{T_{cond}}{T_{cond} - T_1} \]

Energy Quantity

\[ E = \int_S [T_b(t, r, x_0, k_{sb}, \alpha_{sb}) - T_b(t, r, x_0 = 0, k_{sb}, \alpha_{sb})]V(r)ds \]

Conductivity + Convection

Phase Change

Compression

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Correlation between Energy and Hydrodynamic parameter of VGS

\[ E_0 = (h_{inl} - h_{out})\dot{V} = c_p\rho\Delta T\dot{V} = c_p\rho\Delta T\dot{V}_{ave}S \Rightarrow V_{ave} = \frac{E_0}{c_p\rho\Delta T S} \]
Dimensionless governing equations, initial and boundary conditions

**Energy Equations**

\[ \alpha_{sb} \frac{\partial \theta_s}{\partial t} = \frac{\partial}{\partial r} \left( r \frac{\partial \theta_s}{\partial r} \right), \text{ for } r > 1 \]

\[ \frac{\partial \theta_b}{\partial t} + (1 - r^2) \frac{\partial \theta_b}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta_b}{\partial r} \right), \text{ for } r < 1 \]

**Initial Conditions**

\[ \theta_s = \theta_b = 1 \]

at \( t = 0 \) and all \( r \) and \( x \)

**Boundary Conditions**

\[ \theta_s = \theta_b, \text{ for } r = 1, \text{ for all } x \]

\[ \theta_s \to 1 \text{ as } r \to \infty, \text{ for all } x \]

\[ \frac{\partial \theta_b}{\partial r} = k_{sb} \frac{\partial \theta_s}{\partial r}, \text{ for } r = 1, \text{ for all } x \]

\[ \theta_b = 0, \text{ for } x = 0 \text{ and } r < 1 \]
Similarity problems for VGS

All *dimensionless characteristics* of the problem are:
- two parametric
- depend on two similarity parameters

\[ \alpha_{sb} = \frac{\alpha_s}{\alpha_b} \quad \text{and} \quad k_{sb} = \frac{k_s}{k_b} \]

In particular,

\[ L_0 = \varphi(\alpha_{sb}, k_{sb}) \frac{2\dot{V}}{\pi \alpha_b} \]
Example

**BRINE**

30% propylene glycol

and 70% water

\[ k_b = 0.44 \, W/mK, \]

\[ \alpha_b = 1.1 \cdot 10^{-7} \, m^2/s \]

**SOIL**

1. Sand with 20% water content

\[ k_s = 1.33 \, W/mK, \]

\[ \alpha_s = 9.26 \cdot 10^{-7} \, m^2/s \]

2. Granite

\[ k_s = 3.5 \, W/mK, \]

\[ \alpha_s = 1.19 \cdot 10^{-6} \, m^2/s \]

\[ k_{sb} = 3.02, \]

\[ \alpha_{sb} = 8.42 \]

\[ k_{sb} = 7.95, \]

\[ \alpha_{sb} = 10.81 \]
Analysis of results

**Energy Quantity**

\[
E(t, \alpha_{sb}, k_{sb}, x_0) = c_p \rho \int_S [T_b(t, r, x_0, \alpha_{sb}, k_{sb}) - T_b(t, r, x_0 = 0, \alpha_{sb}, k_{sb})]V(r)ds
\]

\[
E^*(t, x_0, \alpha_{sb}, k_{sb}) = \frac{E}{c_p \rho \Delta T V} = 4 \int_0^1 \theta_b(t, r, x_0, \alpha_{sb}, k_{sb})(1 - r^2)rdr
\]

**Energy Quality**

\[
\text{COP}_{\text{Carnot}} = \frac{T_{\text{cond}}}{T_{\text{cond}} - T_1} = \frac{1}{1 - \frac{T_0}{T_{\text{cond}}} + \frac{\Delta T}{\bar{\theta}(t, x_0, \alpha_{sb}, k_{sb}) T_{\text{cond}}}}
\]

\[
\bar{\theta}(t, x_0, \beta, \gamma) = 2 \int_0^1 \theta_b(t, x_0, r, \alpha_{sb}, k_{sb})rdr
\]

\[
\Omega(t, x_0, \alpha_{sb}, k_{sb}) = \{E^*(t, x_0, \alpha_{sb}, k_{sb}), \bar{\theta}(t, x_0, \alpha_{sb}, k_{sb})\}
\]

Capacity of the well

Dimensionless debit of the energy well

Average dimensionless temperature

The fundamental energy characteristic

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Capacity $E^*$ of a borehole heat exchanger (BHE) as a function of its dimensionless length, time and similarity parameters $\alpha_{sb}$, and $k_{sb}$

$E^*$ as a function of $x_0$ for different values of $t$, $k_{sb}$, and $\alpha_{sb}$.

- For granite: $k_{sb} = 7.95$, $\alpha_{sb} = 10.81$
- For sand with 20% water content: $k_{sb} = 3.02$, $\alpha_{sb} = 8.42$
Calculated average dimensionless temperature \( \tilde{\theta}(t, x_0, \beta, \gamma) \) as a function of dimensionless length \( x_0 \), dimensionless time \( t \), and similarity parameters \( \alpha_{sb}, k_{sb} \).

### Diagrams

- **Left Diagram**:
  - \( t = 0.1 \)
  - \( \alpha_{sb} = 10.81 \)
  - \( k_{sb} = 7.95 \)
  - Granite

- **Right Diagram**:
  - \( t = 0.1 \)
  - \( \alpha_{sb} = 8.42 \)
  - \( k_{sb} = 3.02 \)
  - Sand with 20% water content
CONCLUSIONS

- Developed an unsteady phenomenological hydrothermodynamic theory of VGS - an integrated system combining soil, BHE, and GSHP in a nonequilibrium thermodynamic system.

- The introduced fundamental energy characteristic \( \Omega(t, x_0, \alpha_{sb}, k_{sb}) = \{E^*(t, x_0, \alpha_{sb}, k_{sb}), \theta(t, x_0, \alpha_{sb}, k_{sb})\} \) of an energy well for a given VGS completely defines the quantitative and qualitative energy characteristics of a given VGS.

- The results allow for new scientific-technological approaches to creating optimum strategies for the design, control and operation of VGS.
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